

Research Projects [G0M67A]

Greenhouse Effect of Clouds on Exoplanets

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Abstract

Context. Surface temperatures of exoplanets are an important factor when looking for extraterrestrial life, since it determines whether or not liquid water can exist on the planet, paving the way for life. Right now, surface temperatures can not be measured directly for any known exoplanet, hence modeling is required.

Aim. In this project, the goal is to develop a model to computationally study the greenhouse effect caused by exoplanet clouds and atmosphere.

Methods. The model is a 5 layer computational model, which will calculate the Greenhouse factor, which will show whether the Greenhouse effect increases or decreases the equilibrium temperature. This way, it can be determined whether the surface of a planet could conceivably contain liquid water. The constructed model will be tested on a documented planet of our solar system, Venus.

Results. Looking at Venus with only an atmosphere (no clouds), we obtain a Greenhouse factor larger than unity. Hence, the surface temperature of the planet will be higher than the original equilibrium temperature. However, when adding clouds (atmosphere and cloud layers), the Greenhouse factor becomes smaller than unity, hence a cooling effect would occur. This cooling is not observed, Venus is significantly hotter than its equilibrium temperature. Hence this model does not take into account enough factors to determine the overall observed Greenhouse factor. This is to be expected, as this model is a first approximation. To get more accurate results, further development of the model and code to include additional effects is needed.

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1 Introduction

When studying exoplanets, an important factor to consider is its surface temperature. For the purposes of this paper, this is mainly due to its importance in determining the possible existence of liquid water on its surface. As liquid water is considered essential to the formation of life, this can determine whether a planet might be hospitable to it or not.

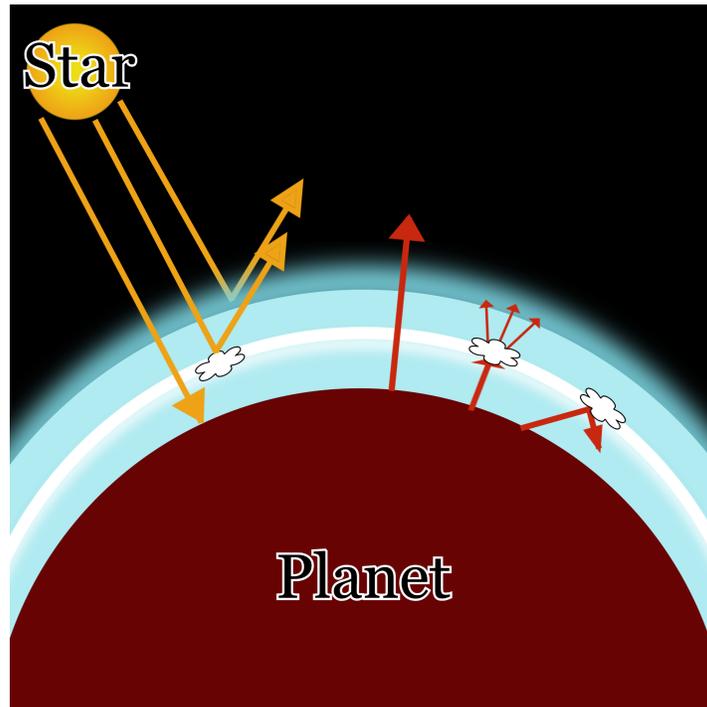


Figure 1: Schematic Greenhouse effect

An important effect to take into account when determining the surface temperature of a planet is the Greenhouse Effect, the process which explains the interaction between the incoming stellar radiation and the complete atmosphere (See fig. 1). The model described below will attempt to make a first approximation computational model of the Greenhouse effect due to the atmosphere and clouds of exoplanets, to determine whether the studied planet will have a surface temperature larger or smaller than the original equilibrium temperature.

2 A Five System Model

2.1 General Overview

In this section (A Five System Model), we describe the theory for the model used in this project. This includes all the main features, simplifications and considerations we took into account in order to obtain a first approximation of the greenhouse effect occurring in exoplanets.

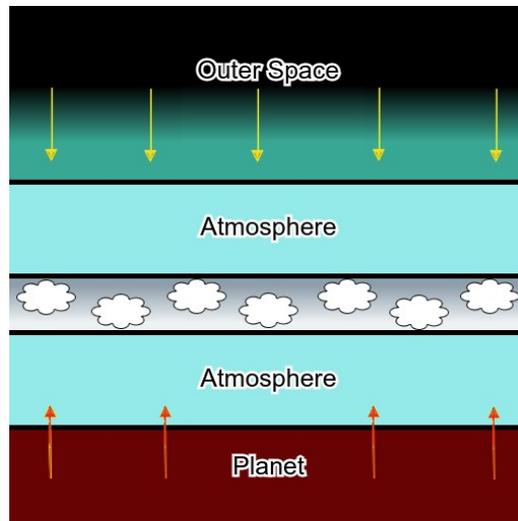


Figure 2: Diagram used model (5 layer)

We assume a five system model composed of a single star (Outer Space), an exoplanet orbiting around its host star (Planet), an upper atmosphere, a cloud layer and the lower atmosphere of the exoplanet (See fig. 2). The main assumptions made are

- Planetary rotation is not considered.
- The planet is spherically symmetric.
- The planet is in constant thermal equilibrium.
- The atmosphere and cloud layers are considered to be flat (plane parallel assumption).
- The only radiative process considered is the absorption in the line of sight.

2.2 Black Body Radiation

Both the star and the planet in our model are approximated as blackbody radiators. Hence, the spectral radiance of the star and planet (emitted over all wavelengths) is only a function of their temperature T and is given by Planck's Blackbody Radiation Law:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_b T}\right) - 1} \quad (1)$$

where λ is the wavelength, $k_b = 1.381 \times 10^{-23}$ J/K is the Boltzmann constant, $h = 6.626 \times 10^{-34}$ J/Hz is the Plank constant and $c = 299,792,458$ m/s is the speed of light.

Our model assumes that the only incoming electromagnetic radiation source - which comes from outer space and reaches the top of the atmosphere of the planet - is its host star. Hence, the incoming radiation, assuming no interaction with matter before reaching the planet, is found to be:

$$B_{\text{Star-to-Planet}}(\lambda, T) = \frac{R_{\text{planet}}^2}{4d^2} B_{\text{star}}(\lambda, T) \quad (2)$$

where d is the distance between the star and the planet.

After integration, the total energy emitted by the planet - which is approximated as a spherically symmetric blackbody - is given by the Stefan-Boltzmann law:

$$L_{\text{planet}}(T) = 4\pi R_{\text{planet}}^2 \sigma T_{\text{planet}}^4 \quad (3)$$

where $\sigma = 5.670374 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$ is the Stefan-Boltzmann constant.

If the planet is in thermal equilibrium, the integral over the incoming spectral radiance and the emitted energy of the planet must be equal:

$$4\pi R_{\text{star}}^2 \sigma T_{\text{star}}^4 \frac{R_{\text{planet}}^2}{4d^2} = 4\pi R_{\text{planet}}^2 \sigma T_{\text{planet}}^4 \quad (4)$$

Thus the planet equilibrium temperature, considering only the incoming stellar radiation, is

determined to be:

$$T_{\text{eq}} = \left(\frac{R_{\text{star}}}{2d_{\text{planet}}} \right)^{1/2} T_{\text{star}} \quad (5)$$

2.3 Radiative Transfer in Atmospheres

In order to describe the effect on each layer when radiation passes through it, we need to consider the radiative transfer equation:

$$\frac{dB(\lambda, T, z)}{dz} = -(\alpha(\lambda, z) + \gamma(\lambda, z))B(\lambda, T, z) \quad (6)$$

where z is the height from the surface of the planet and $\alpha(\lambda, z)$ and $\gamma(\lambda, z)$ the wavelength and height dependent absorption and scattering, respectively. These quantities - α and γ - can be expressed as follows:

$$\alpha(\lambda, z) = \sum_s \sigma_{\text{abs},s}(\lambda) c_s(z) \rho(z) \quad (7)$$

$$\gamma(\lambda, z) = \sum_s \sigma_{\text{sca},s}(\lambda) c_s(z) \rho(z) \quad (8)$$

where

- $\sigma_{\text{abs},s}, \sigma_{\text{sca},s}$ are the absorption and scattering cross sections for each element or molecular species s of which the atmosphere is composed
- c_s the height dependent concentration of s
- ρ the pressure distribution of the planet's atmosphere

Thus, the solution of the radiative transfer equation can be written as:

$$B(\lambda, T, z) = B_0(\lambda, T) \exp \left(- \sum_s (\sigma_{\text{abs},s}(\lambda) + \sigma_{\text{sca},s}(\lambda)) N_s(z) \right) \quad (9)$$

With the column density of a species s defined to be:

$$N_s(z) = \int_{z_0}^z c_s(z')\rho(z')dz' \quad (10)$$

where z_0 is the lower boundary, closest to the surface, where the atmospheric layer starts and z is it's upper boundary.

2.4 Radiative Transfer in Clouds

2.4.1 Mie Theory

Let us assume a molecular or elemental species, composed of particles of the simplest form (spherical and isotropic particles). For a given species - with mean grain radius α - the wavelength dependent extinction cross section σ_{ext} is the sum of the absorption cross section σ_{abs} and the scattering cross section σ_{sca} , that is:

$$\sigma_{ext}(\alpha, \lambda) = \sigma_{abs}(\alpha, \lambda) + \sigma_{sca}(\alpha, \lambda) \quad (11)$$

In order to obtain expressions that are suited for computations using these cross sections (absorption and scattering cross sections), we use Mie Theory. This is a mathematical and physical theory providing quantitative results for both light absorption and scattering by clouds (composed of spherical and isotropic particles). More precisely we use the approach of Bohren - Huffman [3], according to which absorption and scattering cross sections can be written as infinite series expansions:

$$\sigma_{sca}(\alpha, \lambda) = \frac{2\pi\alpha^2}{x^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2) \quad (12)$$

$$\sigma_{abs}(\alpha, \lambda) = \left(\frac{2\pi\alpha^2}{x^2} \sum_{n=1}^{\infty} (2n+1)Re(a_n + b_n) \right) - \sigma_{sca}(\alpha, \lambda) \quad (13)$$

where $x(\alpha, \lambda, N) = 2\pi\alpha N/\lambda$ is the wavelength dependent size parameter and a_n, b_n the scattering coefficients depending on the size parameter and the relative refractive index

$m = N_1/N$. N_1 and N are the refractive indices of the particle and the surrounding medium, respectively. Mie Theory allows simple numerical calculations of the scattering coefficients by using the Riccati - Bessel functions:

$$\psi_n(y) = yj_n(y) \quad (14)$$

$$\xi_n(y) = yh_n^{(1)}(y) \quad (15)$$

where j_n and $h_n^{(1)}$ are Rayleigh's formula and spherical Bessel function of the third kind, respectively. Some formulas that can be used to express these functions are:

$$j_n(y) = (-y)^n \left(\frac{1}{y} \frac{d}{dy} \right)^n \frac{\sin y}{y} \quad (16)$$

$$h_n^{(1)}(y) = j_n(y) - i \left(\frac{1}{y} \frac{d}{dy} \right)^n \frac{\cos y}{y} \quad (17)$$

Thus, for a given n , the scattering coefficients can be written as follows:

$$\alpha_n = \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)} \quad (18)$$

$$b_n = \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)} \quad (19)$$

These expressions include second derivatives of ψ_n and ξ_n . We can eliminate them by using the recurrence relations:

$$\psi_n'(x) = \psi_{n-1}(x) - \frac{n\psi_n(x)}{x} \quad (20)$$

$$\xi_n'(x) = \xi_{n-1}(x) - \frac{n\xi_n(x)}{x} \quad (21)$$

Hence the expressions for the scattering coefficients can be simplified and reformulated as

follows:

$$a_n = \frac{m\psi_n(mx)\psi_{n-1}(x) - \psi_n(x)\psi_{n-1}(mx) - \frac{n}{x}\psi_n(x)\psi_n(mx)\left(m - \frac{1}{m}\right)}{m\psi_n(mx)\xi_{n-1}(x) - \xi_n(x)\psi_{n-1}(mx) - \frac{n}{x}\psi_n(mx)\xi_n(x)\left(m - \frac{1}{m}\right)} \quad (22)$$

$$b_n = \frac{\psi_n(mx)\psi_{n-1}(x) - m\psi_n(x)\psi_{n-1}(mx)}{\psi_n(mx)\xi_{n-1}(x) - m\xi_n(x)\psi_{n-1}(mx)} \quad (23)$$

2.5 The greenhouse Factor

In order to obtain an estimation of the greenhouse effect on the exoplanet, we first assume three distinct systems: the planet, outer space and the system that is composed out of two atmospheric layers and one cloud layer.

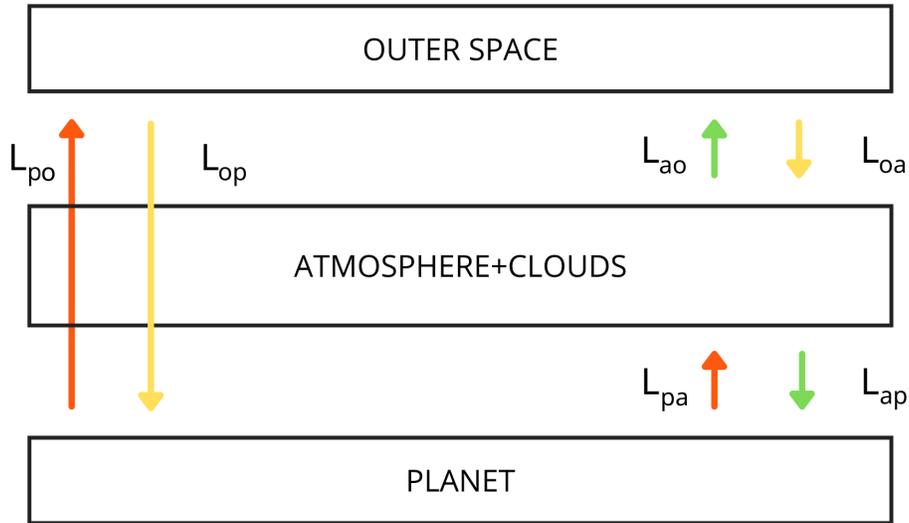


Figure 3: Outer space, Atmosphere+Clouds, Planet in thermal equilibrium

It is considered that these three separate systems each have their own temperature and that they interact solely through radiation. Hence, the equations which describe their thermal equilibrium are given by:

$$L_{ao} + L_{po} - L_{oa} - L_{op} = 0 \quad (24)$$

$$L_{oa} + L_{pa} - L_{ap} - L_{ao} = 0 \quad (25)$$

$$L_{ap} + L_{op} - L_{pa} - L_{po} = 0 \quad (26)$$

where the planet, outer space and the atmosphere - including clouds - are denoted by p, o and a respectively.

We define three quantities: f_{pa} , f_{oa} which express the fraction of energy blocked by the atmosphere that is coming from the planet or outer space respectively, and g which expresses the fraction of energy coming from outer space to reach the planet surface compared to the total incoming energy from outer space. Thus:

$$f_{pa} = \frac{L_{pa}}{L_{pa} + L_{po}} = \frac{L_{pa}}{L_{\text{planet}}} \quad (27)$$

$$f_{oa} = \frac{L_{oa}}{L_{oa} + L_{op}} = \frac{L_{oa}}{L_{\text{space}}} \quad (28)$$

$$g = \frac{L_{ap}}{L_{ap} + L_{ao}} \quad (29)$$

Using these fractions, L_{po} and L_{op} can be written as follows:

$$L_{po} = (1 - f_{pa})L_{\text{planet}} \quad (30)$$

$$L_{op} = (1 - f_{oa})L_{\text{space}} \quad (31)$$

Combining the thermal equilibrium equations and the equations (24)-(30), we can deduce an expression for the total energy coming from the planet, L_{planet} , given by:

$$L_{\text{planet}} = L_{po} + L_{pa} \quad (32)$$

$$= L_{ap} + L_{op} \quad (33)$$

$$= g(L_{pa} + L_{oa}) + (1 - f_{oa})L_{\text{space}} \quad (34)$$

$$= g(f_{pa}L_{\text{planet}} + f_{oa}L_{\text{space}}) + (1 - f_{oa})L_{\text{space}} \quad (35)$$

$$= \frac{1 - f_{oa} + gf_{oa}}{1 - gf_{pa}}L_{\text{space}} \quad (36)$$

Using equation (28), g can also be written in terms of R_{lower} and R_{upper} , the distances of the lower and upper layer of the atmosphere from the planet, respectively. Thus:

$$g = \frac{4\pi R_{\text{lower}}^2}{4\pi(R_{\text{lower}}^2 + R_{\text{upper}}^2)} \quad (37)$$

If we assume a very thin spherical atmosphere, such that $R_{\text{lower}} \approx R_{\text{upper}}$, then $g \approx 0.5$ and the expression (35) for L_{planet} can be reformulated as:

$$L_{\text{planet}} = \frac{1 - 0.5f_{oa}}{1 - 0.5f_{pa}}L_{\text{space}} \quad (38)$$

We can then finally define a greenhouse factor as:

$$G_H = \frac{1 - 0.5f_{oa}}{1 - 0.5f_{pa}} = \frac{L_{\text{planet}}}{L_{\text{space}}} \quad (39)$$

where f_{oa} and f_{pa} can be calculated if we know the total incoming radiation from the host star and the fraction of it which passes through the planet's atmosphere, since:

$$f_{oa} = 1 - \frac{\int_A \int_0^\infty B_{\text{Reaches-ground}}(\lambda, T) d\lambda dA}{\int_A \int_0^\infty B_{\text{outer-space}}(\lambda, T) d\lambda dA} \quad (40)$$

Using eq. (5) we can obtain a formula for the modulated equilibrium temperature of the

planet which includes the greenhouse effects:

$$T_{\text{eq}}^G = \left(\frac{R_{\text{star}} \sqrt{G_H}}{2d_{\text{planet}}} \right)^{1/2} T_{\text{star}} \quad (41)$$

Thus:

- if $f_{pa} > f_{oa}$, that is if the atmosphere which includes the clouds are such that the radiation coming from the planet is stronger absorbed than the incident stellar radiation, then, by (38), $G_H > 1$, and thus, by (40), an increase of the equilibrium temperature of the planet is expected. This occurs when the atmosphere blocks more radiation at wavelength bands in which the planet radiates.
- if $f_{pa} < f_{oa}$, that is if the atmosphere which includes the clouds are such that the incident stellar radiation is stronger absorbed than the radiation coming from the planet, then, by (38), $G_H < 1$, and thus, by (40), an increase of the equilibrium temperature of the planet is expected. This occurs when the atmosphere is transparent in a wavelength band at which the planet radiates.
- if $f_{pa} = f_{oa}$, that is if the atmosphere which includes the clouds are such that the amount of energy received by the atmosphere is half due to planetary radiation and half due to stellar radiation, then, by (38), $G_H = 1$, and thus, by (40), and no greenhouse effect occurs on the planet.

3 Results and Discussion

In order to properly test the model, a planet with a relatively simple atmospheric structure has to be used. As Venus' atmosphere is documented by multiple space missions and is known to consist mainly of CO_2 , it would be a suitable first candidate. It's black body temperature is taken to be 329K.

3.1 A simple Venus Atmosphere

A first thing to consider when studying an atmosphere is its composition. The NASA Venus data sheet [8] shows that the atmosphere of Venus has a CO_2 abundance of 96.5%. Hence, a first estimate would be to consider the atmosphere of Venus to consist completely of CO_2 .

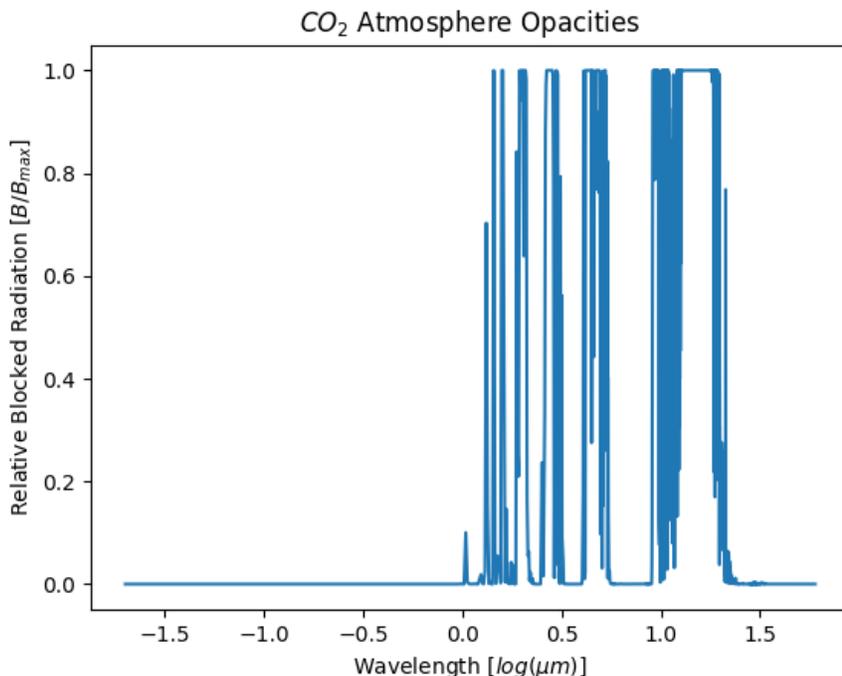


Figure 4: Atmosphere Opacity result [4] [6]

Using data from [6] (number densities atmosphere Venus) and [4] (Absorption Cross sections CO_2) the opacity - which is defined to be the relative blocked radiation - of the atmosphere,

can be determined (see fig. 4). The data models a Venusian atmosphere out to a radius of 380km. The figure shows no absorption below the low wavelength optical region and considerable absorption in the infrared. As a planets emits mainly in the infrared, CO_2 can be seen to be a potent greenhouse gas.

It is important to note that the term "blocked" here does not necessarily mean that the light does not interact with the atmosphere, it just does not reach the surface of the planet. It is possible that it interacts with the atmosphere, causing it not to reach the surface.

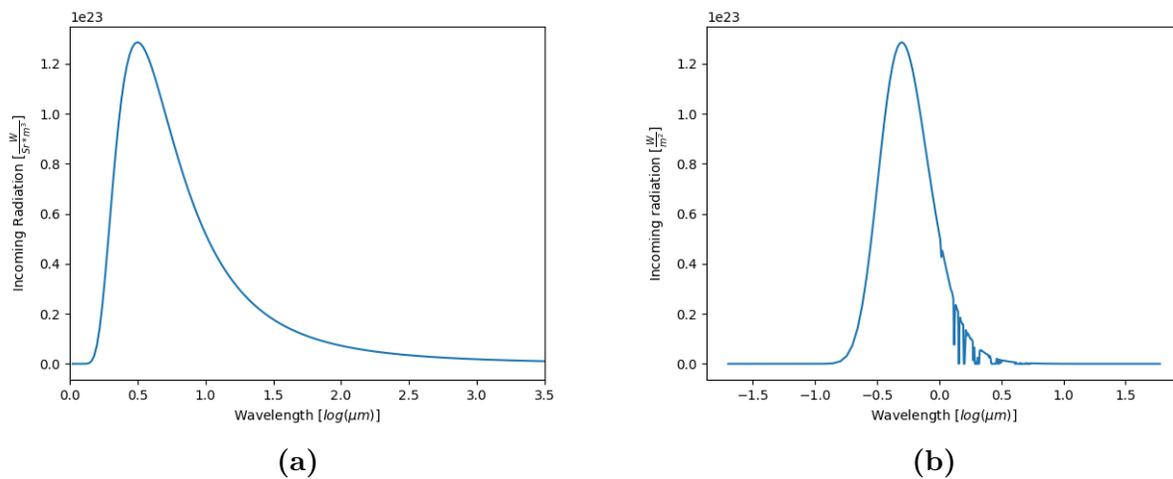


Figure 5: (a) The radiation spectrum from the star before entering the upper atmosphere. (b) The radiation spectrum of the star after passing through the atmosphere.

As mentioned earlier, the resulting incoming radiation as shown in fig. 5a and fig. 5b is barely affected by the CO_2 atmosphere, as CO_2 barely absorbs at the low wavelengths of the stellar spectrum.

Conversely, as can be seen from fig. 6, the outgoing radiation is greatly affected by the CO_2 atmosphere. As the atmosphere consists of only one molecule, the individual CO_2 absorption bands are clearly visible.

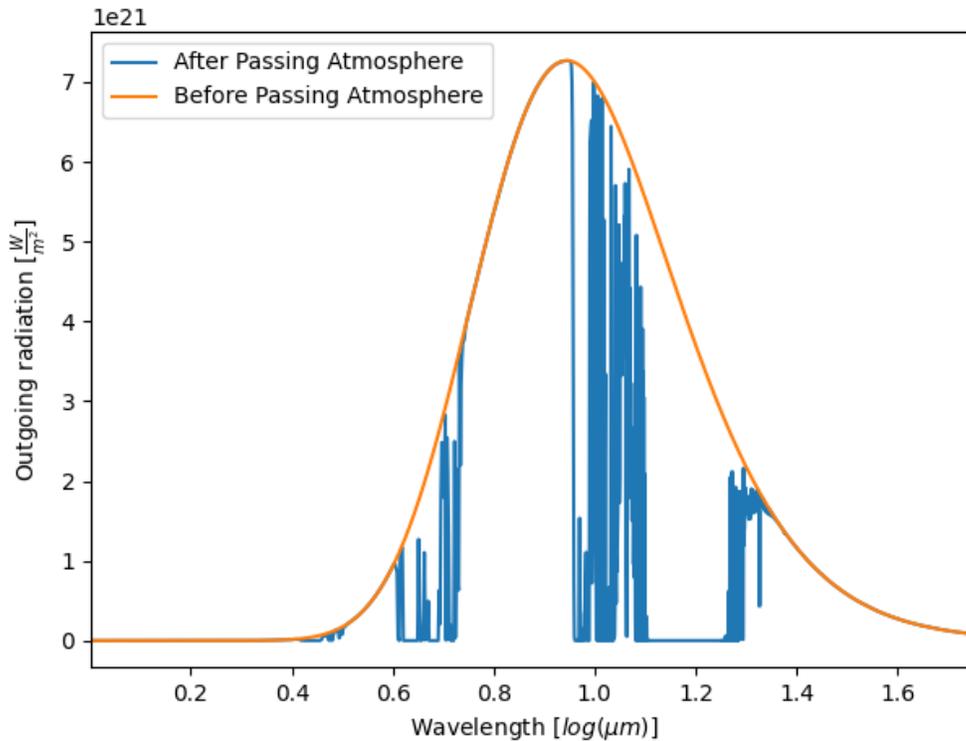


Figure 6: Venus outgoing radiation before and after passing through the CO_2 atmosphere layer

As described in section 2.5, both the incoming and outgoing spectrum can be used to calculate the f-factors, resulting in a greenhouse factor G_H , which determines the relative heating or cooling of the planet. As the incoming radiation is much less affected by the CO_2 , we find a greenhouse factor $G_H = 1.277$, showing an overall heating. The new effective temperature adjusted for this value is then determined by eq. (5) to be about 6.3% higher than the original value, from 329K to 350K.

3.2 Homogeneous H_2SO_4 Clouds

Now the atmospheric opacities are known, the same has to be done for the clouds. The clouds consist mainly of H_2SO_4 and hence, as a first approximation, it is allowed to assume they consist completely of H_2SO_4 .

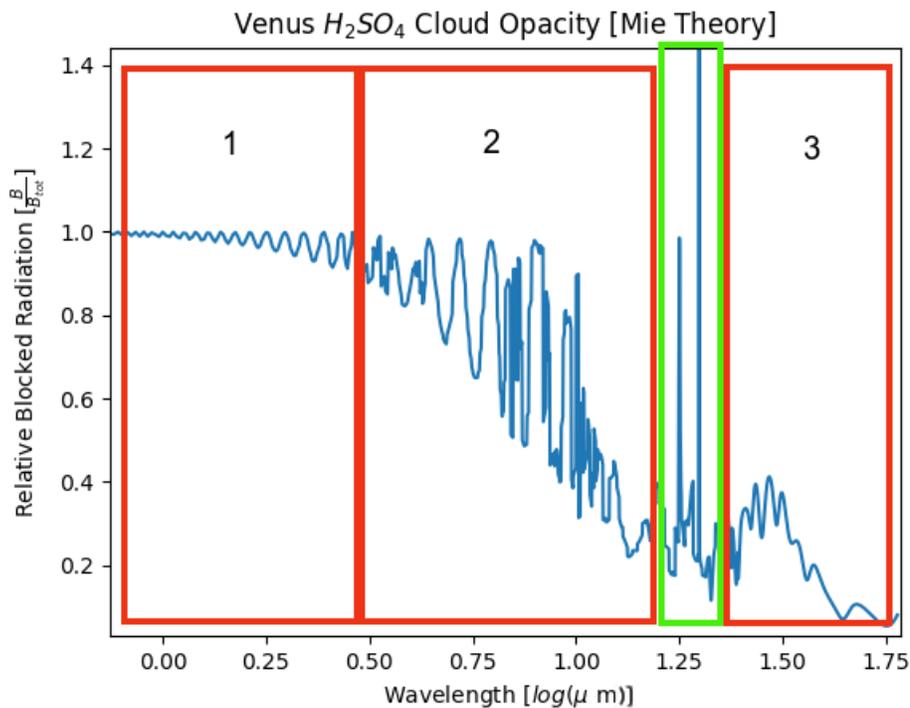


Figure 7: Cloud Opacity result from Mie Theory [2][1]

Calculating the opacities can be done using Mie theory (discussed in section 2.4.1). The data used to calculate the opacities comes from [2] (Cloud structure of Venus: the number densities) and [1] (refractive indices Sulphuric acid). The clouds are inserted into the same atmosphere as the previous section, at a height of 45km to 65km. The resulting opacities can be seen in fig. 7. When looking at this figure, a number of interesting phenomena can be seen. The general structure of the opacity curve can be divided into three main parts.

- The first part is relatively constant, where the relative blocked radiation is approximately equal to 1. This can be interpreted as the wavelength of the incoming radiation being so small that the particles behave as brick walls for it: nothing can pass through.
- The second part of the curve shows a downward trend, indicating that as the wavelength increases, the probability that the light gets blocked becomes smaller because the relative size of the particle gets smaller as compared to the wavelength.
- In the third part the cloud opacity tends to zero as the wavelength becomes even larger. This is because the cloud particle size gets small compared to the wavelength of the incoming light. At some point this relative particle size gets so small that the incoming radiation almost isn't affected by the clouds at all.

The last major feature of importance are the peaks as seen in the green box. These peaks are typical for Mie theory and they do not necessarily have a physical interpretation. These are resonance peaks which are created when the wavelength approaches the size of the effective radius of the cloud particles.

3.3 Resulting Effects

Now that the opacities for both clouds and atmosphere have been calculated, the results can be combined to construct the 5 layer model (see fig. 2). First the effect of the atmosphere and clouds is calculated for the incoming star light. (see fig. 8a and fig. 8b).

The incoming radiation is just a blackbody spectrum from the host star, scaled according to eq. (2). (fig. 8a). After passing through the complete atmosphere and clouds, the resulting "arriving" radiation is given by fig. 8b. It can be seen that the blackbody curve has been reduced considerably. Most of the reduction is due to the clouds, as they mainly block the low wavelength light coming from the star.

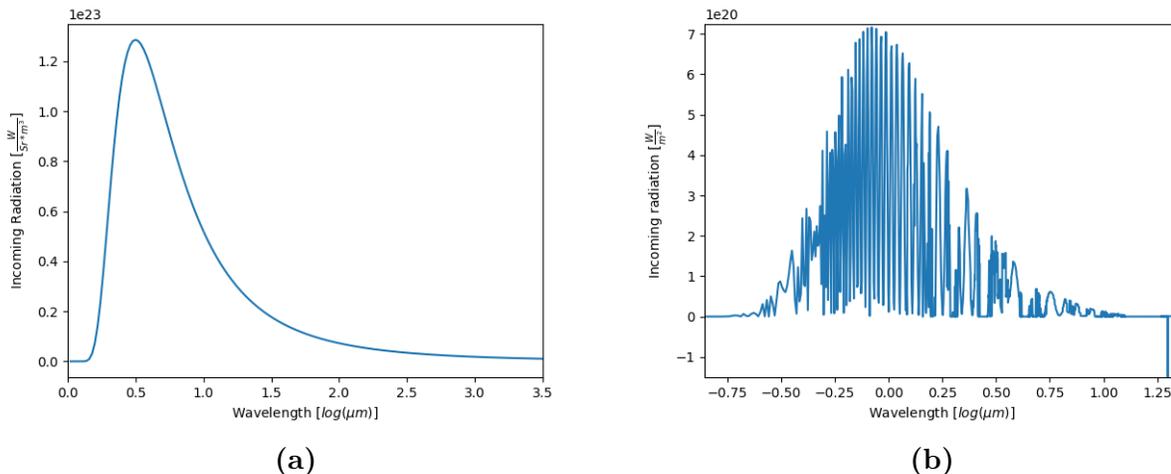


Figure 8: (a) The radiation spectrum from the star before entering the upper atmosphere. (b) The radiation spectrum of the star after passing through both layers of the atmosphere and clouds.

One thing that stands out is the negative incoming radiation peak. When looking in detail, it can be seen that this peak corresponds to the same wavelength as the resonance peak in the Mie theory result for the cloud opacity. Hence the resonance peak in Mie theory causes this unphysical "negative incoming radiation".

Now the effect of the atmosphere and clouds is calculated for the outgoing planet radiation (fig. 9).

The outgoing radiation is just a blackbody spectrum from the planet. After passing through the complete atmosphere and clouds, the resulting "Leaving" radiation is smaller than the original blackbody spectrum (see fig. 9). It can be seen that the "perfect" blackbody shape - as for the star - has been deformed and hence at certain wavelength ranges, radiation has been blocked/absorbed.

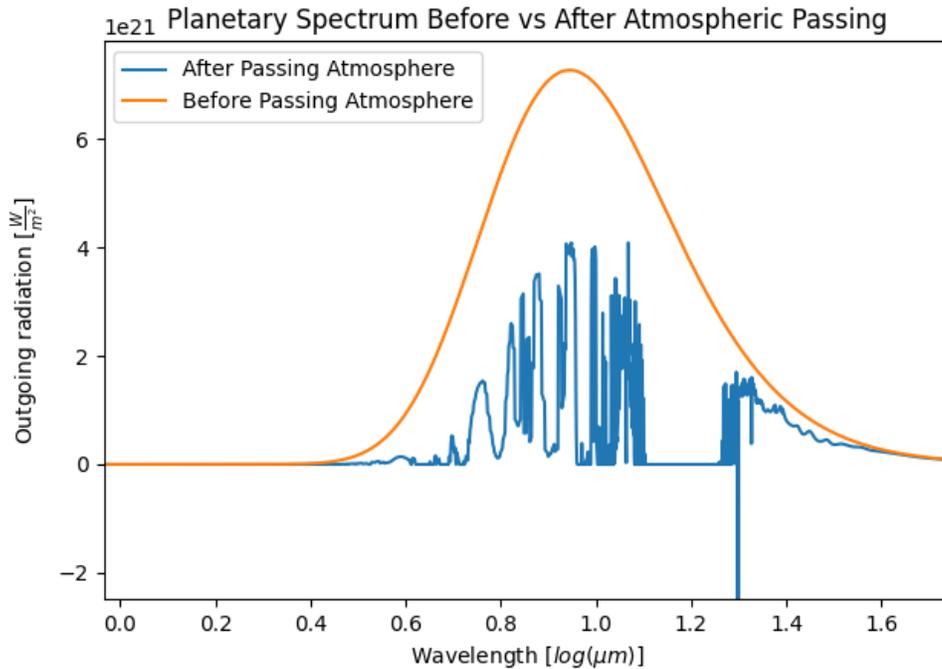


Figure 9: Venus outgoing radiation before and after passing through the atmosphere and cloud layers

Because of the planet's lower temperature, the peak of the outgoing spectrum lies in the infrared. Since the absorption due to Mie theory is less pronounced here, the reduction is also less severe than for the incoming radiation spectrum. A significant amount of absorption thus happens due to the influence of the CO_2 atmosphere. Similarly as for the incoming radiation, a negative outgoing radiation peak due to Mie theory can be seen.

If we repeat the same calculation as in section 3.1, we obtain a greenhouse factor $G_H = 0.814$, resulting in an overall reduction in temperature of 5%, decreasing the temperature from 329K to 312K. As such, the H_2SO_4 clouds, while thick, appears to have a cooling effect on its effective temperature. This shows that the significantly higher atmosphere temperature we observe can be attributed to other physical processes.

3.4 Observational Difficulties and Data Acquisition

Surface temperatures can not be measured for any known exoplanet with currently available instruments. The surface temperatures depend on the equilibrium temperature of the planet and the greenhouse effect. There are observational results that indicate that Earth-like planets orbit other stars relatively often. Since they could potentially have a composition similar to Earth's, they are interesting targets to study when looking for habitable environments outside the solar system. However, solely using experimental methods to determine the characteristics of the planet is quite difficult for such small planets, and hence models requiring limited data are needed.

However, modeling the surface temperature of an exoplanet is difficult since many quantities (physical and chemical) that determine the exoplanet surface properties are (currently) not measurable [7] [5]. Because of this, the initial goal was to test the model on our own planet, Earth. However, since the composition of the atmosphere and cloud layers of earth are complex, it was then decided to choose one of our neighbouring planets in the solar system, Venus. Venus has a relatively simple atmospheric composition and clouds to model as compared to Earth.

However, finding data (number densities, refractive indices, ...) for the atmosphere and clouds for Venus was very difficult, but eventually proved doable. Since this data acquisition (for such a "close" planet) was already difficult, one can imagine that finding the necessary exoplanet data can prove to be even more of a challenge, especially when looking for earth-like exoplanets (as described above in this section).

4 Conclusion

In this project, a five layer computational model is developed to study the Greenhouse effect due to the atmosphere and clouds of exoplanets. The goal of the model is to calculate the Greenhouse factor, which will determine whether the planet will be hotter or cooler than its equilibrium temperature. This computational model is a first estimate and makes a lot of assumptions.

The atmosphere used to test the model was that of Venus, which to first approximation consists of a CO_2 atmosphere and H_2SO_4 clouds. The opacities of both the atmosphere and clouds were successfully calculated, where Mie theory was used for the clouds. All expected features of Mie theory could be observed, including the resonance peaks.

The change in the radiation spectra - of the star and planet - when having passed the atmosphere and cloud layers were also successfully calculated. The spectra generally became smaller, but the effects of the clouds and atmosphere separately could also be observed and were as expected. Adding a cloud layer (atmosphere and cloud layers) reduced the spectra even more at many different wavelengths, when being compared to the case when no cloud layer (only atmosphere) was present in the calculation.

The Greenhouse factor was successfully calculated for two cases: One case with solely a CO_2 atmosphere and one with a CO_2 atmosphere and H_2SO_4 clouds:

- CO_2 atmosphere: Greenhouse factor = 1.277
- CO_2 atmosphere and H_2SO_4 clouds: Greenhouse factor = 0.814

Hence in our model Venus would be hotter than its equilibrium temperature when solely having a CO_2 atmosphere, while it would cool down when having a CO_2 atmosphere and H_2SO_4 clouds. The cooling down of Venus is an unexpected result, since Venus is a very hot planet. Hence the model does not take into account all factors needed to calculate the Greenhouse Effect precisely. This of course is an expected result since this model is a first approximation, and hence further development of the model is needed.

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